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Designing Tasks to Examine Mathematical Knowledge for Teaching Statistics for Primary Teachers

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Abstract. Mathematical knowledge for teaching (MKT) is viewed as fuel resources for conducting an orchestra in a teaching and learning process. By understanding MKT, especially for primary teachers, it can predict the success of a goal of an instruction and analyze the weaknesses and improvements of it. To explore what teachers think about subject matters, pedagogical terms, and appropriate curriculum, it needs a task which can be identified the teachers' MKT including the subject matter knowledge (SMK) and pedagogical content knowledge (PCK). This study aims to design an appropriate task for exploring primary teachers' MKT for statistics in primary school. We designed six tasks to examine 40 primary teachers' MKT, of which each respectively represents the categories of SMK (common content knowledge (CCK) and specialised content knowledge (SCK)) and PCK (knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC)). While MKT has much attention of numbers of scholars, we consider knowledge of content and culture (KCCI) to be hypothesized in the domains of MKT. Thus, we added one more task examining how the primary teachers used their knowledge of content (KC) regarding to MKT in statistics. Some examples of the teachers' responses on the tasks are discussed and some refinements of MKT task in statistics for primary teachers are suggested.

1. Introduction

In this recent decade, there has been a growing research interest in the notion of knowledge that mathematics teachers need to master in order to make their teaching effective. This is primarily originated from the work of Shulman [1] which was then modified by other researchers such as Ball et al [2] for their mathematical knowledge for teaching (MKT). In particular, the research in the investigation of statistical knowledge for teaching has also obtained a greater attention by scholars. The attention is based on common reports which revealed that better teaching of statistics requires better training for the teachers. As with no specific training, they are likely to fall back on what are often erroneous intuitions and beliefs [3].

It is important, therefore, to get some prior studies by examining primary teachers' competencies regarding basic statistical knowledge, particularly when it comes to basic concepts such as a central tendency covering mean, median, modus, which is included in typical mathematics curriculum for primary students. To that, some have concerned on exploring teachers' knowledge of subject matter on statistics [4, 5,6], while others concerned on pedagogical content knowledge on statistics (PCK) [7, 8].



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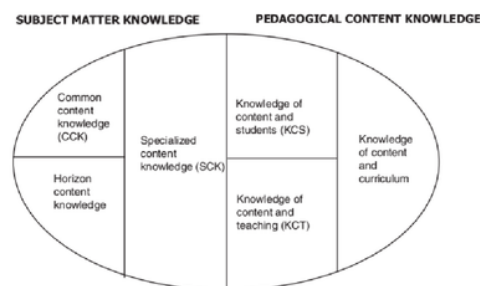


Figure 1. Domains of Mathematical Knowledge for Teaching [2]

MKT domain proposed by Ball *et al* [2] as shown in figure 1 encompasses the commonly expected mathematical knowledge of adults as well as the specialised mathematical knowledge of the classroom needed by teachers to carry out the “work of teaching mathematics” [2]. They categorized the domain into two types namely subject matter knowledge (SMK) and pedagogical content knowledge (PCK). SMK consists of common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK), while PCK consists of knowledge of content and student (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC).

In SMK, Ball *et al* [2] defines CCK as the mathematical knowledge and skill used in settings other than teaching. Teachers need to know the topic they teach such as they must recognize an inaccurate definition [28]. In short, SCK is defined as a mathematical knowledge and a unique skill in teaching. For example, it requires understanding certain interpretations of the mean in ways that students should figure out, such as inferencing mean from a fixed total [24], appreciating the difference between models of mean [2], and distinguishing interpretation of mean as data reduction, fair share, typical value, and signal in noise [10].

With regard to PCK, Ball *et al* [2] defines KCS, as the knowledge combining students and mathematics knowledge. Teachers must anticipate what students are likely to think and what they will find confusing. In coping with the concept of mean, for example, teachers need to understand the common errors that students make when solving a mean-related problem, or figuring out the students’ difficulties and misconceptions in the concept of mean. KCT, on the other hand, is defined as the knowledge that combines knowledge in teaching and mathematics. For instance, teachers need to understand which representation they should use to introduce the concept of mean, what sequence of problems they should use to begin the work on the concept of mean, or how chronologically arrange the solution methods that should be presented in a whole class discussion. In knowledge of content and curriculum (KCC), Ball *et al* [2] actually is not yet sure whether this may be a part of their category of knowledge of content and teaching or whether it may run across several categories. However, they refer KCC to Shulman’s [1] initial argument which defines knowledge of curriculum as a set of programs designed for teaching certain subjects and topics at particular level, the diversity of available teaching materials related to such programs and a number of characteristics that are both indications and contra indications use in a specific curriculum [1]. To cope with the topic of mean at primary level, for example, teachers need to understand how to determine learning goals about the topic of mean for a particular activity or what prior knowledge related to mean that primary students should know.

However, we found limited resources that examined teachers’ MKT both in SMK and in PCK in a statistics concept, particularly in the topic of mean for primary teachers. Whereas previous studies, as we found, only examined SMK [4] and PCK [8] separately. In addition, there has been only a limited research on PCK for statistical education. Sosa [11] finds that the majority of studies addresses the knowledge of the content (KC) to be taught (about 88%), while less than a half study specified PCK or knowledge regarding to student learning process. Thus, in this present study, we designed a set of tasks that examine MKT in a such topic and discussed how primary teachers worked on the MKT tasks. Here, our concern is MKT in central tendency topic, particularly, ‘mean’ taught at primary school level. In addition, MKT encompasses CCK, SCK, KCS, KCT, and KCC. We also add one more task about how culture is introduced in teachers’ teaching knowledge and practice. This task discusses a potential

teachers' knowledge we hypothesized as one of the teachers' MKT namely knowledge of content and culture (KCCl).

2. Method

In designing MKT tasks for the statistic subject matter for primary teachers, we were guided by some theoretical pieces of literature, mainly by the works conducted by Shulman [1] and Ball et al. [2] and the findings which are relevant to MKT for central tendency. In coping with CCK, we considered referring the relevant theory from Ortiz and Font [4] which examined teachers' understanding on the estimation of an unknown quantity. For SCK, we considered the model of task developed by Zaskis [9] which examined teachers' understanding of a mean interpretation inferred from a fixed total. Furthermore, KCS concerned on examining teachers' knowledge of students' common errors and difficulties in a mean-related problem, while KCT and KCC respectively concerned on teachers' knowledge about how to teach relevant representations under the topic of mean and the order of prior knowledge taught before learning mean. One additional task regarding to KCCl was added as our hypothesis (see Table 1).

To confirm the validity of the MKT tasks, we asked an expert to validate the questions by giving constructive suggestions regarding content and language use. To discuss teachers' MKT on the concept of mean, we analyzed forty primary teachers' responses on the MKT tasks which exemplify the representatives of various responses found in all MKT types.

Table 1. MKT Tasks for Statistics at Primary Level

No	MKT types	Questions	Focus concern	MKT Test
18				7. Definitions [2]
1	Common content knowledge	Eight students from a class weigh a bag of sugar using the same instrument, with the following values in grams being obtained: 6.2, 6.0, 17 6.3, 6.1, 6.23, 6.15, and 6.12. What would be the best estimate of the sugar's real weight?	Estimation of an unknown quantity	Knowing the facts and concepts of the discipline of mathematics, i.e. mean
2	Specialized content knowledge	The weight of an object made from clay is 91.1 grams. The object was split into twelve smaller objects with different weight. What was the value of the average weight? Explain your strategy	Focus on inference from a fixed total	Presenting explanations for certain procedures
3	Knowledge of content and students	Imagine your students are asked to solve this problem. "A set of data from eight students' math scores give mean, median, and mode respectively of 7, 8, and 7.2. Find three possible sets of those eight scores which fit the requirement." What errors and difficulties which may you probably find from your students' work? Explain.	Common errors and difficulties regarding the concept of mean, median, and mode in a problem-solving task	Anticipating student difficulties and errors when presented with a mathematical task
4	Knowledge of content and teaching	Which representations (figures, learning media, visual display, etc) would you use to introduce the concept of mean? How would you use such representations in your teaching?	Models of representation and its teaching sequence of mean	7. Selecting models, representations and procedures that support the development of mathematical understandings
5	Knowledge of content and curriculum	a. What topics before 'mean, median, and mode' that primary students should understand? b. Do you think all primary graders (from 1 to 6) are able to learn about mean, median, and mode? If Yes, How? If No, Why?	Learning trajectory on mean, median, and mode for primary students	Knowing what should be taught regarding central tendency in order

6	Knowledge of Content and Culture*	a. Which local wisdom in Indonesia would you think is appropriate with the topic of mean, mean, and mode? b. How would you bring that local wisdom into your teaching practice of mean, median, and mode?	Local wisdom on mean, median, and mode	-
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*hypothesized

3. Results

3.1. Teachers' SMK on the concept of mean

We obtained four categories of CCK regarding to the estimation of an unknown quantity. Category 1 corresponds the responses that indicate the use of the arithmetic mean and the term 'mean' explicitly. Category 2 corresponds the indication of the use of arithmetic mean but does not indicate the term 'mean' explicitly. At last, category 3 portrays those which do not indicate the use of arithmetic mean and the term 'mean' explicitly. To grab overall pictures, Figure 2 shows clarifies the four categories mentioned.

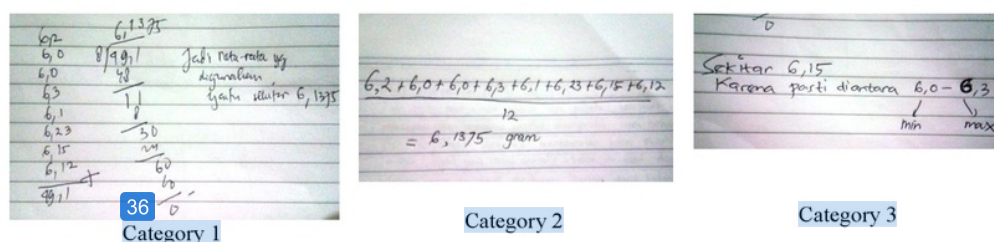


Figure 2. Teachers' CCK responses

Regarding to the teachers' SCK, we found three categories of responses namely (1) correct responses which indicate the use of inference from a fixed total, (2) correct responses which indicate the use of a strictly applied algorithm, and (3) incorrect responses. The incorrect ones were due to the miscalculation when applying the mean algorithm. Figure 3 illustrates the examples of the first two categories.

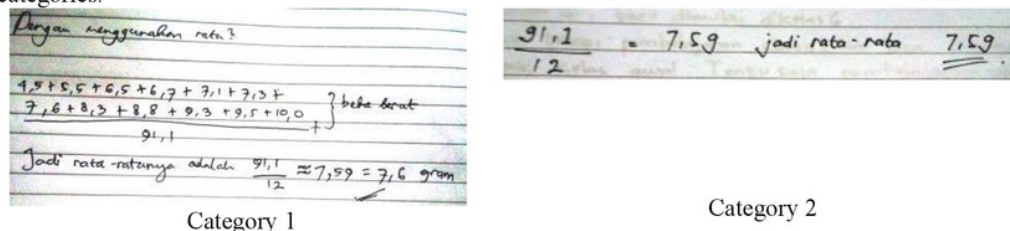


Figure 3. Examples of SCK in Category 1 and 2

Category 1 shows that the teacher likely did not really understand the concept of mean from the perspective of the inference from a fixed total. The teacher added up all the data and divided the sum by numbers of data, which means the teacher tried to first list a set of possible weight of clay which is different each other and applied the standard arithmetical mean. In contrast, category 2 indicates a teacher's understanding in a fixed total of mean concept, indicated by the teachers' direct calculations; dividing the total of data, instead of listing a possible set of different data, by the number of objects.

3.2. Teachers' PCK on the concept of mean

Discussing KCS, we divide the teachers' responses into two types namely difficulties and errors. There are three errors that students experience, such as they (1) do not know that median should be in between the fourth and the fifth datum when the data has been in order, (2) do not think that mode is the only one that can be found from the set, and (3) know how to make the sets of data but create incorrect calculation. Meanwhile, we found two categories of student difficulties namely (1) finding more than a set of

possible data and (2) determining which one of mean, median, or mode that should be considered as the easiest way to use a mathematical formula. One of the teachers' responses based on the latter difficulty is given as follows.

"When selecting mean first, students will likely think of the total data, obtaining $8 \times 7 = 56$, and make eight data whose sum, median, and mode respectively is 56, 8, 7.2. When selecting mode, they have already understood that the mode, which is 7.2, must be contained in the set of data. Thus, 7.2 must be most frequently found from the set. I think the best way is by starting from the mode, the median, and finally, the mean, because mode is always found in a set of data"

The response above shows that the teacher knows that the errors regarding the misunderstanding of the mode and median could lead to the student difficulties in selecting what should be firstly considered for any calculation.

Regarding KCT, we obtained two types of representations the teachers proposed for teaching mean concept: figural and physical activities. The figural representations, as shown in figure 4, were selected to be introduced in the teachers' teaching sequences. The idea of teaching using such representations is that teachers would start from giving a set of similar objects, i.e. candies, marbles, small stones, balls or other familiar objects. The students would get the different number of objects. The students' task is to think of how to share their objects each other so that every student has the same number of the objects. Meanwhile, the physical activities were suggested to be used for learning mean concept by having students to do some measurement activities, such as measuring students' height or weight, then finding its average using a mean formula.

Comparing those two types of representations, we argue that the figural representations are closer to the expected model of teaching which is more constructive, attractive, and interactive since students should discuss with their peers how to share the objects and to think about 'a number' which represents the same number of objects for each student. However, either the figural representations or the physical activities the teachers offered seemed to have students learned only about the 'meaning of mean', instead of the 'concept of mean', whereas many educators suggest to set learning sequences which aim students [17] for examples, make sense with other central tendency topics such as median and mode, identify that average does not necessarily equal with one of the values summed [12] or various types of mean interpretations as described by Konold & Pollatsek [10].

With regard to KCC, the teachers responded that students should understand about ordering numbers, a basic arithmetical operation involving decimals, and extreme values in a set of data. When asked to argue whether it is possible for all primary graders to learn about the concept mean, median, mode, we found there were two categories namely agree and disagree statements. Those who agreed stated that actually, students at lower grades have learned about basic statistics such as counting objects based on certain representations, determining which object has the largest, the smallest, and the most middle number (median) and pointing the most common height out from the students (mode). However, those who disagreed argued that mean, median, and mode can only be taught at upper primary grade, since their complexities when using formula and involving non-integer number as well as interpreting word problems related to those three topics.

3.3. Teachers' KCCI on the concept of mean

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Our proposal regarding KCCI was originated from an idea proposing that differences between students' and teachers' cultural backgrounds and teachers' limited knowledge of the students' cultures present challenges for teachers to plan and to teach in ways that harness on their students' background experiences [13]. In MKT, therefore, teachers are encouraged to map the cultural knowledge around students' environment, such as local wisdom, cultural and natural heritages, which potentially affect on students' understanding of a certain topic in mathematics and, then, finally apply the mappings into their instructional practices. For example, when teaching 'mean' at grade six, teachers can bring the folklore found at the place where students are schooling. See [14].

The teachers in this study answered the KCCI task with various responses, ranging from giving examples of using local *batik* (traditional cloth), traditional song, dances, and games, to the increase of

visitors in a temple. The idea of using those examples is by asking students to obtain data showing the trend of a certain data which is related to, for examples, the numbers of batik home industries or the traditional games preferred by students. However, we found no teachers responded by showing how MKT are explicitly integrated into the local wisdom they mentioned. The only responses we found is that there was a response explained about how to use a traditional game, called *congklak*, to introduce mean, median, and mode. But, it seems limited to explore why and how to bring the games into teaching practices.

4. Discussion and Conclusion

In our MKT tasks, we designed all the MKT types from the teacher participants particularly on the topic of mean-related concepts. However, we suggest that the MKT tasks can still be improved regarding the content of focus concern included in the set of MKT tasks. For example, further research can give additional CCK referred to Ortiz and Font used Batanero's statistical problem to examine teachers' CCK [4] such as about obtaining equal shares in order to achieve a uniformed distribution, finding an element that represents a set of given values whose distribution is approximately symmetrically framed by a context being one of data comparison, concerning a situation on which the task is to determine the value that is most likely to be obtained when selecting a random element from a population; and about the weighted arithmetic mean. Moreover in PCK, further designers can also consider the [4] K on statistics of Watson et al.'s which consists of four aspects: recognizing big idea, anticipating student answers, employing content-specific strategies, and constructing shifts to general [8]

To conclude, we would highlight that the design of MKT task for statistics in primary school resulted. We suggest completing the MKT tasks with the characteristics of MKT component as well as central tendency concepts. This is to develop a more comprehensive task designed to examine teachers' MKT on central tendency, instead of mean, median, mode, separately. In addition, further studies may also use an experimental method to examine the validity and the reliability of the MKT tasks.

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